Predicting the Onset of Instabilities in Frequency Multipliers

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Abstract - Microwave multipliers are notoriously unstable with input power, bias and load variations. In this paper, we predict the onset of a chaotic type of instability using the Hopf bifurcation concept as the input power level is increased. The measurement and the predicted theoretical results are in agreement to within 1.5 dB.

INTRODUCTION

Frequency multipliers are notorious for their parametric instabilites. No work has been done on predicting such instabilities accurately. In this paper, we predict using the Hopf bifurcation concept the onset of spurious oscillations which lead to a chaotic type of instability in a microwave multiplier. Previous stability-related work using bifurcation concepts have been carried out in frequency divider and oscillator circuits[1-4]. For purpose of demonstration, a varactor doubler based on a pn junction diode nonlinearity is chosen to be representative of a multiplier. The doubler is seen to exhibit a spurious oscillation route to chaos beyond a critical value of the input power level . The oscillations are a consequence of the dynamical negative resistance manifested by the pn junction diode. The circuit is designed to convert the frequency from 5 GHz to 10 GHz.

ANALYSIS

The analysis of the multiplier is based on the piecewise harmonic balance method [5] in which the circuit is separated into nonlinear and linear parts as shown in Figure 1. The stability analysis follows the procedure in [6] but the formulation given here is believed simpler and still applicable to practical microwave circuits. Without loss of generality, a representative nonlinear device can be modeled as a parallel combination of nonlinear resistor and a nonlinear capacitor. The Y-parameters of the linear portion

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of the circuit is obtained using the microwave simulator MDS [7]. Then following [8-9], the harmonic balance equation for the one-port circuit is given as

$$E(V_k) = I_{g,k} + I_{c,k} + Y(k\omega_o)\cdot V_k \qquad (1)$$

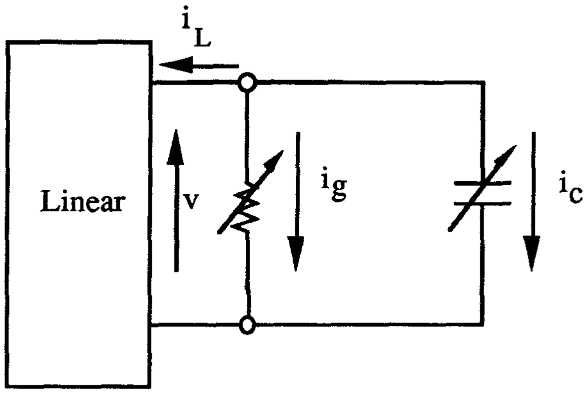

Figure 1: A typical linear-nonlinear network configuration for multipliers.

The perturbed equation is in the form

$$\sum_k \sum_l J_{k-l}\cdot\Delta V_k = 0 \qquad (2)$$

where

$$J_{k-l} = \left[G_{k-l} + \left(s+\Omega_{k,l}\right)C_{k-l} + Y\left(\Omega_{k,l} - js\right)\right] \qquad (3)$$

represents the Jacobian of (1) for s=0 (unperturbed state), and is used in the Newton-Raphson method for solving (1). This Jacobian consists of the nonlinear conductance matrix, the nonlinear capacitance matrix, and the admittance matrix of the linear circuit. Following the Nyquist approach in [6], the determinant of (3) modified by a factor of $\exp(-\pi s/\omega_o)$ need to be plotted only in the range $[0,\omega_o/2]$ to extract the stability information. The Nyquist criterion for instability becomes

$$N_o = Z; \;\Rightarrow\; \text{stable if and only if } N_o = 0 \qquad (4)$$

where N_o is the total number of encirclements of the origin, and Z is the number of unstable zeros on the right-hand plane (RHP) of the s-plane.

The bifurcation parameter (input power) is stepped gradually up from a low level. At each solution point determined by the harmonic balance analysis, the local stability at that point is found by using the Nyquist approach. In this way, a large signal stability chart is computed.

RESULTS

Figure 2 depicts the theoretical and experimental curves of the output power of the frequency doubler at 10 GHz with the bias level held fixed at -2.0 V. The experimental Hopf bifurcation point observed at the input power level of 11 dBm confirms well with the theoretical estimate of 10.4 dBm. At the bifurcation point, a pair of zeros crosses over the imaginary axis to the RHP. However, the magnitude of the self-oscillation is very small. Hence, the harmonic balance and stability analysis based on the single-tone spectrum is still valid, and it approximates the experimental curve well beyond the initial Hopf point to 12 dBm. At this input power level, a second pair of zeros crosses over to the RHP and the magnitude of the multi-tone spectral lines increases dramatically. The measured output power becomes considerably higher than the calculated curve. A likely cause is that the negative resistance of the diode increases and acts as a parametric amplifier to the input signal. Since this is not taken into account in this single-tone analysis, the experimental curve is higher than the numerical curve. To construct the numerical curve accurately, it is necessary to resort to a complete multi-tone harmonic balance analysis. This is especially true close to the 12 dBm point since the mixing products become considerably larger.

CONCLUSION

We have predicted the arrival of spurious oscillation which leads to chaos in a multiplier using the Hopf bifurcation concept, and have confirmed experimentally the validity of our formulation. This finding helps the circuit designer to locate the instability points for input power variation.

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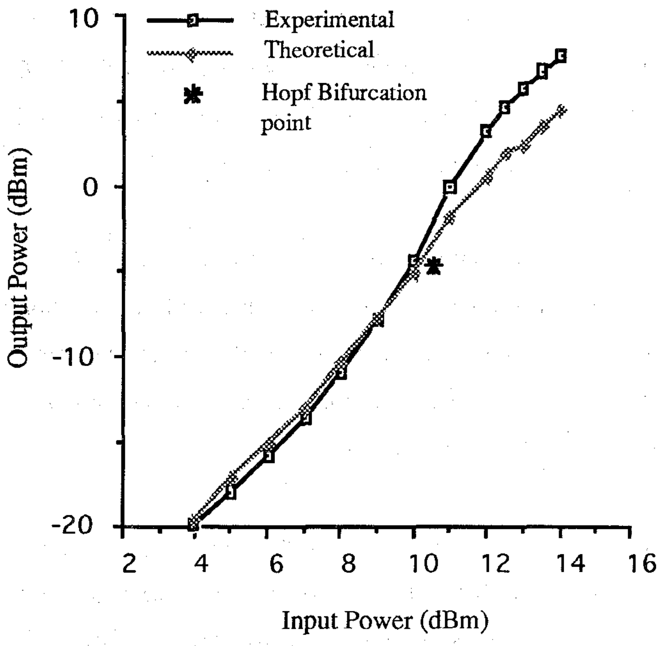

Figure 2:Theoretical and measured output power of doubler and location of Hopf bifurcation point with input power variation.